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A note on accelerating cosmologies from compactifications and S-branes

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Abstract: We give a simple interpretation of the recent solutions for cosmologies with a transient accelerating phase obtained from compactification in hyperbolic manifolds, or from S-brane solutions of string/M-theory. In the four-dimensional picture, these solutions correspond to bouncing the radion field off its exponential potential. Acceleration occurs at the turning point, when the radion stops and the potential energy momentarily dominates. The virtues and limitations of these approaches become quite transparent in this interpretation.

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There is a current effort to understand the recently discovered cosmic acceleration, as well as to embed the inflation paradigm, within a fundamental framework such as string/M-theory. Progress in this direction has been hampered by some generic arguments, which are often regarded as a no-go theorem, that forbid cosmic acceleration in the cosmological solutions obtained from compactifications of any pure supergravity model [1]. In more detail, one assumes that the internal manifold is compact, static, and non-singular.

In recent work, Townsend and Wohlfarth have pointed out that it is possible to find solutions that exhibit a transient phase of acceleration if the size of the internal manifold is allowed to change in time [2]. Further extensions of this idea have appeared more recently in [3]–[5], where it is argued that a similar behavior is present as well in certain time-dependent solutions of supergravity with p-form fluxes, known as S(pacelike)-branes [6]–[8]. Earlier work on closely related cosmologies can be found in [9, 10].

Our purpose in this note is to point out that the qualitative properties of these solutions become particularly transparent by going to a reduced four-dimensional description. In this framework, the solutions are viewed as cosmologies with scalar exponential potentials, and the origin of the accelerating phase admits a simple and illuminating interpretation.

The essence of the idea is as follows. The construction of these models involves compactification on a maximally symmetric space, plus possibly the fluxes of covariantly constant four-form field strengths. In the reduced description, all of these give rise to exponential potential terms for the compactification volume scalar ψ, also known as the "radion". For sufficiently large negative ψ, the potential is a positive exponential $V(\psi) \sim +e^{-\alpha\psi}$, while for $\psi \to +\infty$, the potential goes to zero as $V(\psi) \sim \pm e^{-\beta\psi}$. Suppose now that the field starts at a large value of $\psi > 0$, with very large negative velocity $\dot{\psi}$. This corresponds to a kinetic-dominated regime with equation of state $p \simeq \rho$. So the universe starts out in a phase of decelerated expansion. If the initial kinetic energy of ψ is large enough, then at a later time we will find the field running up the exponential potential hill $+e^{-\alpha\psi}$. Eventually it will stop, due to the increase in potential energy and to cosmic friction. So around this point, the potential energy dominates: the universe enters an accelerating phase. Soon after this, the field starts rolling back down the hill, gaining kinetic energy again and entering a new decelerating phase. The endpoint of the evolution depends on the details of the potential — typically the field rolls back down to $\psi \to \infty$, but depending on whether the potential is asymptotically positive or negative, there will be a slowly decelerating expansion, or collapse. These are the attractor solutions of the system.

This kind of behavior is precisely the one exhibited by the solutions of [2]. It is then clear that the acceleration is all the result of initial conditions where the field starts out with a large kinetic energy, so it can climb up the positive exponential potential and attain a brief interval of potential domination around the turning point. The expansion that one can get in this way seems rather modest. On the other hand, the exponential potentials that arise from either compactification or four-form fluxes are not enough to yield power-law inflation. Therefore it seems just as difficult as before to obtain a sufficient number of e-foldings of accelerated expansion in this class of cosmologies.

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Let us now flesh out the previous discussion with some calculations. We take our higher-dimensional lagrangian to be

$$I = \frac{1}{16\pi G_{4+n}} \int d^{4+n}x\sqrt{-G}\left(R[G] - \frac{1}{2\times 4!}F_{[4]}^2\right), \tag{1}$$

and let the geometry be a warped product of a four-dimensional spacetime and an internal compact space $\Sigma_{\sigma,n}$ which we shall assume to be maximally symmetric, with $R_{ab}(\Sigma_{\sigma,n}) = \sigma(n-1)g_{ab}$, $\sigma = -1, 0, +1$,

$$ds^2 = e^{-n\psi(x)}g_{\mu\nu}(x)dx^\mu dx^\nu + e^{2\psi(x)}d\Sigma_{\sigma,n}^2\,. \tag{2}$$

The field strength is taken as $*F_{[4]} = b\,\mathrm{vol}(\Sigma_{\sigma,n})$ (we consider $b > 0$ without loss of generality). The factor $e^{-n\psi(x)}$ in front of the four-metric $g_{\mu\nu}$ has been chosen so that $g_{\mu\nu}$ is the Einstein metric in four dimensions. Indeed, upon the dimensional reduction we get [11]

$$I = \frac{1}{16\pi G_4}\int d^4x\sqrt{-g}\left(R[g] - \frac{n(n+2)}{2}(\partial\psi)^2 - V(\psi)\right), \tag{3}$$

where

$$V(\psi) = \frac{b^2}{2}\,e^{-3n\psi} - \sigma n(n-1)e^{-(n+2)\psi}\,. \tag{4}$$

A case of particular interest is $n = 7$ since it corresponds to a compactification of eleven-dimensional supergravity on $\Sigma_{\sigma,7}$. For a vacuum solution, $b = 0$, as considered in [2], we see that in order to have a positive potential the compact manifold has to be hyperbolic, $\sigma = -1$ [12]. Allowing for a non-vanishing flux b always gives a positive exponential term.

The potential (4) is of the sort we described in the introduction. The different possibilities are plotted in figure 1. One can now use the intuition gained from the study of cosmologies driven by scalar potentials to understand qualitatively the possibilities permitted by time-dependent compactifications of (1). For example, all cases with positive exponential potentials will be qualitatively quite similar. So hyperbolic compactifications, with or without fluxes, and flat compactifications with fluxes, will exhibit similar behavior, and all of them will admit a phase of accelerated expansion. Details such as the rate of acceleration can change depending on where in the potential the field comes to a halt. Also, the late time behavior will be different, since the attractor depends on the sign and exponent of the dominating exponential term for large ψ. But it is easy to infer the qualitative features of the solution from the general properties of $V(\psi)$.

The case that introduces more complications is that of spherical compactification. If $b = 0$ the potential is a negative exponential, and clearly there is no possibility of an accelerating phase. When $b \neq 0$ there is a competition between a positive and a negative exponential, and there appears a wider variety of possible behaviors. However, acceleration is clearly possible for solutions where ψ has a turning point at positive values of $V(\psi)$.

We can check the correctness of this qualitative discussion using the time-dependent solutions of the lagrangian (1) found in [6].[1] The ansatz used there for the solutions of

[1]It is possible to consider as well a dilaton ϕ, coupling as $e^{-a\phi}F_{[4]}^2$. This has the virtue that it allows to recover the solutions of [2] from the ones in [6] with $\sigma = -1$ by simply letting $a \to \infty$. As we explained, all hyperbolic compactifications, with our without flux, are qualitatively similar.

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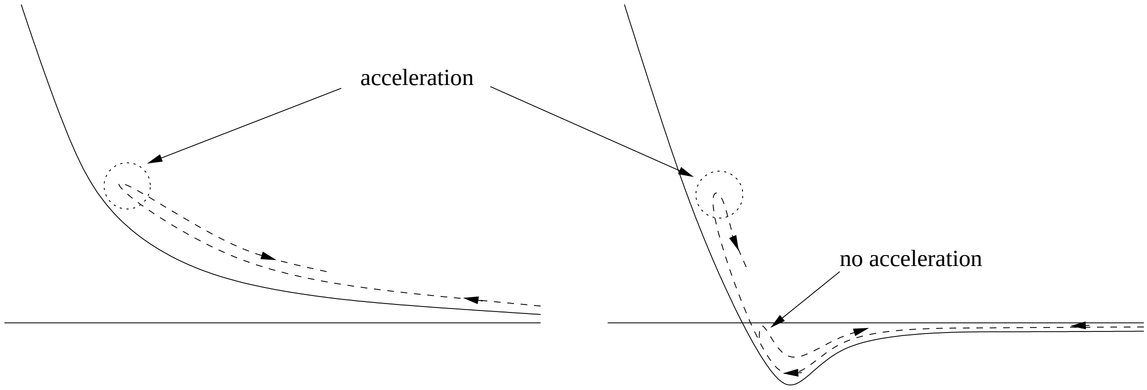

Figure 1: Motion of the radion ψ in the potential $V(\psi)$. In the left figure all the exponentials in the potential are positive. The field starts out at $\psi \to +\infty$ with very large (infinite) kinetic energy. Around the point where it turns around, the energy is potential-dominated and acceleration occurs. In the right figure there is a combination of a positive exponential (coming from four-form flux) and a negative exponential (from spherical compactification). We show two trajectories: one reaches $V(\psi) > 0$ and hence produces positive acceleration (although the universe may be expanding or collapsing). The other turns around at $V = 0$ and therefore does not lead to acceleration. All these possibilities are realized in the solutions described in the text. The asymptotic future behavior is determined by the attractor solutions of each potential.

relevance to us is

$$ds^2 = -e^{6B+2nC}dt^2 + e^{2B}d\mathbf{x}^2_{(3)} + e^{2C}d\Sigma^2_{\sigma,n}\,, \tag{5}$$

with B and C functions of t only. We can translate the results of [6] into our formalism by identifying $\psi = C$, and taking the four-dimensional Einstein metric

$$g_{\mu\nu}(x)dx^\mu dx^\nu = -e^{3(2B+nC)}dt^2 + e^{2B+nC}d\mathbf{x}^2_{(3)}\,, \tag{6}$$

which describes a flat FLRW universe with scale factor

$$a(t) = e^{B+nC/2}\,. \tag{7}$$

Proper time (for a four-dimensional observer) is measured by η, obtained from

$$d\eta = a(t)^3 dt\,, \tag{8}$$

and the condition for acceleration is $d^2a/d\eta^2 > 0$.

The solutions are

$$B(t) = -\frac{1}{3}\log\left(b\sqrt{\frac{n-1}{6(n+2)}}\cosh 3(t-t_0)\right), \tag{9}$$

and

$$\psi(t) = C(t) = g(t) - \frac{3}{n-1}B(t)\,, \tag{10}$$

where we have defined

$$g(t) = \begin{cases} \frac{1}{n-1}\log\left(\frac{\beta}{\sinh[(n-1)\beta|t|]}\right), & \sigma=-1\,, \\ \beta t, & \sigma = 0\,, \\ \frac{1}{n-1}\log\left(\frac{\beta}{\cosh[(n-1)\beta t]}\right), & \sigma=+1\,, \end{cases} \tag{11}$$

and

$$\beta = \frac{1}{n-1}\sqrt{\frac{3(n+2)}{n}} \tag{12}$$

(for later use observe that $(n-1)\beta < 3$ when $n > 1$). We have eliminated some of the parameters in [6] via coordinate redefinitions, and have also chosen others so that we obtain the class of accelerating cosmologies. The time shift t_0 does not make any qualitative difference when $\sigma = 0, -1$, but it is important for the spherical case $\sigma = +1$. Note further that hyperbolic compactifications, $\sigma = -1$, present a singularity at $t = 0$. As in [2], in this case we restrict ourselves to $t < 0$. Spacetime is future-complete since $t = 0^-$ corresponds to $\eta \to \infty$. For $\sigma = 0, +1$, t runs from $-\infty$ to $+\infty$. In all cases $t \to -\infty$ corresponds to $\eta \to 0$, and this is interpreted as the big-bang singularity.

It is now a simple task to study the behavior of the volume scalar ψ in these solutions. For comparison with the extensive literature on exponential potentials (see e.g. [13] and references therein), it is useful to write

$$V(\psi) \sim e^{-\lambda\hat{\psi}}\,, \tag{13}$$

where $\hat{\psi} = [n(n+2)/2]^{1/2}\psi$ is the (dimensionless) canonically normalized field. The nature of the attractors depends on whether $\lambda^2 > 6$ ("steep potentials") or $\lambda^2 < 6$ ("flatter potentials"). The flux term in (4) is "steep", $6 \leq \lambda_b^2 < 18$ and positive, whereas the curvature term is "flatter", $2 < \lambda_\sigma^2 \leq 4$, and can have either sign.

At the big-bang, $t \to -\infty$,

$$\psi \to \left(\frac{3}{n-1} - \beta\right)(-t) \to +\infty\,, \tag{14}$$

for all b and σ, and

$$\frac{d\psi}{d\eta} \to -\left(\frac{3}{n-1} - \beta\right) e^{\frac{3}{2}\left(n\beta - \frac{n+2}{n-1}\right)(-t)} \to -\infty\,. \tag{15}$$

Clearly, this is a kinetic-dominated regime, with limiting $p = \rho$. This is an unstable critical point for all types of exponential potentials, so the subsequent evolution flows away from it.

For the evolution at later times we have to consider separately each value of σ:

- For hyperbolic compactifications, $\sigma = -1$, the asymptotic future is at $t \to 0^-$, and then

$$\psi \to \frac{1}{n-1}\log\frac{1}{|t|} \to +\infty\,, \tag{16}$$

so the field rolls back down the hill, now with asymptotic velocity

$$\frac{d\psi}{d\eta} \to |t|^{\frac{3n}{2(n-1)}}\frac{1}{n-1}\log\frac{1}{|t|} \to 0^+\,. \tag{17}$$

This corresponds to the stable attractor of the potential $e^{-(n+2)\psi}$, where the potential and kinetic energies are comparable. In the notation introduced around eq. (13), such attractor exists only for flatter potentials with $\lambda^2 < 6$. The effective equation of state in them is given by $p = [(\lambda^2/3) - 1]\rho$, where $\lambda^2 = \lambda_\sigma^2 = 2(n+2)/n$, and the scale factor behaves as

$$a(\eta) \sim \eta^r \,, \tag{18}$$

where $r = 2/\lambda_\sigma^2 = n/(n+2) < 1$. This is a slowly decelerating, expanding universe. But, obviously, before reaching this regime the field must have turned around, so that at some finite value of $t < 0$ the field halts, $\dot{\psi} = 0$. This happens when

$$\beta(n-1) \coth \beta(n-1)|t| = 3 \tanh 3|t| \,. \tag{19}$$

This transcendental equation has a unique solution for $t < 0$. At around this instant we expect to find a phase of accelerated expansion. This can be easily verified by plotting the evolution of ψ together with the acceleration of the scale factor.

• For flat compactification, $\sigma = 0$, we expect a similar behavior of the accelerating phase, and this expectation is indeed borne out by a closer examination. Asymptotic future is $t \to +\infty$, and in this case $\psi \to (\beta + 3/(n-1))t \to \infty$. As before, there is an instant of finite t where $\dot{\psi} = 0$, and around it the expansion of the universe accelerates.

In contrast to the previous case, the asymptotic future evolution is now kinetic-dominated. This is due to the fact that when $\sigma = 0$, the potential is controlled by the 'flux term' in (4) $\sim e^{-3n\psi}$, which is steeper than the internal volume factor $\sim e^{-(n+2)\psi}$, absent in this case. For steep positive potentials ($\lambda^2 > 6$), the kinetic-dominated expansion is precisely the expected late-time attractor. So this is another example of how the main features of the solutions can be anticipated easily by a direct examination of the potential.

• Finally, for spherical compactifications, $\sigma = +1$, the asymptotic future is different from the previous ones, but in a way that could be anticipated from the attractor of the negative exponential. For all values of the parameters, the above solutions end up in a phase where $\psi \to \infty$ and the universe is described by the collapsing attractor.

To see whether there is a phase of acceleration, we need to take into account the value of the parameter t_0. If we set it to zero, then the solutions are time-symmetric around $t = 0$. This is also the instant where the field then turns around. At this moment,

$$\psi(t=0) = \frac{1}{n-1} \log \frac{b}{\sqrt{2n(n-1)}} \,. \tag{20}$$

Precisely for this value of ψ the potential vanishes, $V(\psi(t=0)) = 0$. Hence there is never a phase of positive potential energy domination, and we do not expect any acceleration in this case. Examination of $d^2a/d\eta^2$ shows this to be the case. At this turning point the universe changes from expansion to collapse.

However, if we allow for $t_0 \neq 0$, the moment when expansion yields to collapse is not an instant of time symmetry. Moreover, the field overshoots and does reach positive values of V, so there is acceleration. If $t_0 > 0$, then the acceleration occurs when the universe is already collapsing, and acts for too short to be able to revert the contraction. However,

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if $t_0 < 0$ the acceleration happens when the universe is still expanding, so we obtain a brief phase of accelerated expansion. These details are hard to discover from a simple examination of the potential, but nevertheless they are at least easily understood within this context.

During expansion, the motion of the field is damped by cosmological friction. On the other hand, during a collapsing phase, the motion of the field is anti-damped, and one might wonder if this might not send the field up the flux-term potential towards $\psi \to -\infty$, causing yet a different type of behaviour without a turning point. It turns out, however, that steep positive potentials with $\lambda^2 > 6$ do not allow such behaviour. Even with the help of anti-friction the field is unable to climb indefinitely and has to turn around.

Our aim here has not been so much the detailed study of the S-brane solutions of [6], but rather to stress that the main features of the cosmologies that one can obtain is best understood using quite conventional ideas of four-dimensional cosmology. In this context, it is worth recalling the possible uses of the accelerating phases described above.

First of all, from solutions of the type given in [2], it appears quite difficult to achieve the sustained period of acceleration needed in inflationary cosmology. An inflationary attractor would require $r > 1$ in eq. (18), or $\lambda^2 < 2$, too flat for the exponentials in (4).

On the other hand, for applications to late time acceleration, where ψ might play the role of a quintessence field, the number of e-foldings required is just of order one. However, the acceleration found in the bounce period is not adequate for this purpose since we know that our universe was not kinetic-dominated before entering accelerated expansion. Finding an appropriate phase of acceleration with the exponential terms in (4) seems unlikely. The approach to the attractor occurs during the radiation and matter dominated eras [14], and by the time the quintessence potential $V(\psi)$ becomes cosmologically relevant the solution is well described by (18). So again, acceleration would need $\lambda^2 < 2$. Furthermore, a quintessential field must be very light, behaving as massless on scales smaller than the Hubble radius. Although the radion ψ is light at large values, it cannot be used for quintessence, since it would mediate long-range interactions of gravitational strength. These are subject to strong observational constraints.

There are several obvious extensions. Compactification on an Einstein manifold, with a reduction truncated to a single volume scalar, leads to the same kind of four-dimensional lagrangians we have studied. One might also consider compactification on product spaces. In these cases one might want to include a volume scalar for each of the components. The resulting potential $V(\psi_i)$ would contain sums and products of exponentials, which may lead to more useful cosmological behaviour. This is currently under investigation.

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